Section 5.4

Definition of the Natural Exponential Function: The inverse function of the natural logarithmic function $f(x) = \ln x$ is called the **natural exponential function** and is denoted by

$$f^{-1}(x) = e^x.$$

That is,

$$y = e^x \quad \text{if and only if} \quad x = \ln y.$$

Operations with Exponential Functions: Let a and b be any real numbers.

1. $e^a e^b = e^{a+b}$ **2.** $\frac{e^a}{e^b} = e^{a-b}$

Derivatives of the Natural Exponential Function: Let u be a differentiable function of x.

1. $\frac{d}{dx}[e^x] = e^x$ **2.** $\frac{d}{dx}[e^u] = e^u \frac{du}{dx}$

Integration Rules for Exponential Functions: Let u be a differentiable function of x.

1. $\int e^x dx = e^x + C$ **2.** $\int e^u du = e^u + C$

1) Solve $3 = e^{4x+3}$.

2) Solve $\ln(5x + 1) = 3$.

3) Differentiate the following:

a) $f(x) = 6e^{3x+4}$

b) $g(x) = e^{\sin x/x}$

4) Find the following:

a) $\int e^{2x-9} dx$

b) $\int 4x^2 e^{-x^3} dx$

c) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

d) $\int \csc^2 x \, e^{\cot x} dx$

5) Evaluate each definite integral.

a) $\int_1^2 e^{-2x} dx$

b) $\int_0^1 \frac{e^x - 1}{e^x} dx$

c) $\int_0^1 \frac{1+e^x}{x+e^x} dx$

Homework for 5.4: #9, 41, 47, 53, 59, 63